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A LIGHT WING†

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In a development of the results obtained previously in [1] a version of the design scheme for a low-mass wing is proposed. It takes the form of a system of spars with each of which a system of cantilevers is linked which participate in the formation of the upper and lower surfaces of the wing. Here, the axes of the spars and the rods forming the wing surfaces are arranged in such a way that there are no torques in the cross-sections of these elements. The dimensions of the sections of each element are chosen so as to ensure equal strength. It is shown that, in the case of this wing scheme, the flexural vibration of the wing which randomly arise does not develop into bending-torsional vibration and does not lead to flutter. The mass of a wing for a light aircraft is estimated. © 1999 Elsevier Science Ltd. All rights reserved.

1. DESCRIPTION OF THE WING

We consider a wing of rectangular shape in plan view which takes up a distributed load (Fig. 1)

$$p(x,y)=p_1(x)p_2(y),\quad x\in[0,b],\quad y\in[0,l]$$

where p_1 and p_2 are specified continuous functions, the first of which characterizes the load distribution over the profile and the second characterizes the load distribution over the wing span in the calculated case. The wing is fixed in the plane $y = 0$. Suppose it is subdivided into m elements and each ith of them ($i = 1, \ldots, m$) consists of a spar with an annular cross-section which is fixed on the fuselage as a cantilever and upper and lower strips of wing skin which are fastened to the spar (Fig. 2) and take up the load confined between the planes $x = x_i$ and $x = x_{i+1}$. Here, $x_i < x_{i+1}$, $x_1 = 0$, $x_{m+1} = b$. The upper strip of a wing skin linked with the spar i is subdivided into m pairs of girders and each kth of them ($k = 1, \ldots, m_i$) consists of a rod of annular cross-section (Fig. 3), which is joined in a cantilever manner to a spar and to a strip fixed to a rod from above which forms part of the upper surface of the wing between the planes $y = y_k$ and $y = y_{k+1}$, where $y_k < y_{k+1}$. In a similar way, the lower strip of wing skin linked with the spar i is divided into cantilevers, each of which has the form of a rod of annular cross-section with a strip joined to it from below.

Suppose the axis of any spar i is specified in the plane $z = 0$ by the line $x = x_i^c$, which obeys the condition that there are no torques in the spar sections

$$\int_{x_i}^{x_i^c} p_1(x)(x_i^c-x)dx=\int_{x_i^c}^{x_{i+1}} p_1(x)(x-x_i^c)dx \tag{1.1}$$

We assume that the change in the external radius of each spar along its axis is described by the function $f(y)$, $y\in[0,l]$. The bending moment in a section of the spar i, which passes through the point (x_i^c, y) of its axis, obviously has the form

$$M_i(y)=I_iI(y);$$
$$I_i=\int_{x_i}^{x_{i+1}} p_1(x)dx,\quad I(y)=\int_y^l p_2(\bar{y})(\bar{y}-y)d\bar{y} \tag{1.2}$$

The stability condition [2] is

$$\sigma J_i(y)\geqslant M_i(y)f(y) \tag{1.3}$$

where σ is the breaking point and $J_i(y)$ is the moment of inertia of the section. Since in order to reduce the wing mass, it is advantageous to increase the number of spars m [1], on decreasing the thickness of the annular cross-section of a spar in this case, the moment of inertia of the cross-section of the spar i about the point (x_i^c, y) of its axis can be taken in the form [2]

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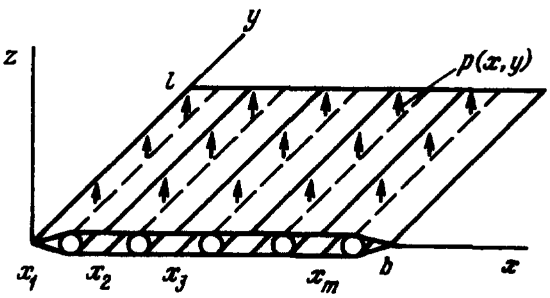

Fig. 1.

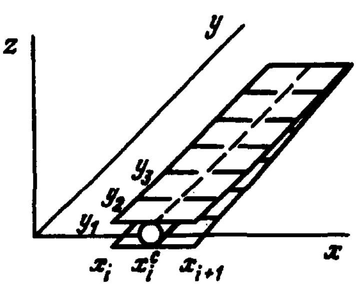

Fig. 2.

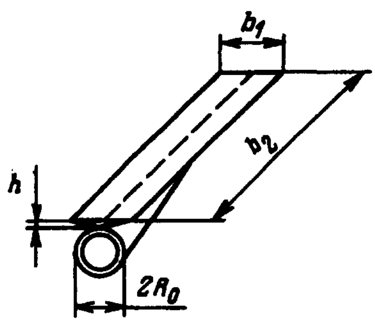

Fig. 3.

$$J_i(y) = \pi f^3(y)\delta_i(y)$$

where $\delta_i(y)$ is the thickness of the annular cross-section of this spar which is chosen, as we propose, such that the stability condition (1.3) takes the form of an equality.

We shall assume that the external radius $f(y)$ of the spar cross-section is several times smaller than the length $(l-y)$ of the cantilever part of the spar corresponding to this section. When account is taken of this, it can be shown for the basic types of load distribution such as, for example, when this distribution is close to a uniform distribution ($p_2(y)$ = const), that the most dangerous points of the section as regards stability are those points where the normal stresses are a maximum and the shear stresses are zero. With this in mind, we shall neglect the effect of shear stresses, which are caused by shearing forces, on the stability.

We shall henceforth use the stability condition (1.3) in the form of an equality. Using this condition, the area of cross-section of a spar is written in the form

$$F_i(y) = 2M_i(y)/[\sigma f(y)] \tag{1.4}$$

and the total mass of the spars, when account is taken of formulae (1.2), is expressed as

$$G^* = \frac{2\rho P_1}{\sigma}\int_0^l \frac{I(y)}{f(y)}dy, \quad P_1 = \int_0^b p_1(x)dx \tag{1.5}$$

where ρ is the density of the wing material. If the spars are conical: $f(y) = f(0)(1-y/l)$ and it is assumed that $p_2(y)$ = const, then

$$G^* = \frac{\rho P l^2}{2\sigma f(0)}; \quad P = P_1 P_2, \quad P_2 = \int_0^l p_2(y)dy \tag{1.6}$$

where P is the overall load on the wing.

We now consider the wing skin elements which are fixed to a spar. We shall assume that the pressure drop across the upper surface of the wing is described by the product $p_1^0(x)p_2(y)$, and across the lower surface of the wing is described by the product $p_1^1(x)p_2(y)$, where p_1^0 and p_1^1 are continuous functions. The upper strip of wing skin linked with the spar i includes two series of cantilevers. Each cantilever k

consists of a rod with a strip fixed to it which takes up the load confined between the planes $y = y_k$ and $y = y_{k+1}$. The axis of the rod and the line of fixing of the strip are arranged in the $y = y_k^c$ plane, where the parameter y_k^c satisfies the condition that there are no torques in the cross-sections of the rod which is analogous to condition (1.1) and is obtained from it by replacing x by y, the function p_1 by p_2 and the subscript i by the subscript k.

A change in the external radius of any rod which is fixed to the spar i as viewed from the spar j is described by the function $f_{ij}(x)$. Moreover, $x \in [x_i, x_i^c]$ when $j = 0$, and $x \in [x_i^c, x_{i+1}]$ when $j = 1$.

The total mass of the rods which are fastened to the spar i as viewed from the spar j to form the upper surface of the wing is expressed in a similar way to the first formula of (1.5) and, if it is assumed that the rods are conical, that is

$$f_{ij}(x) = R_0(x - x_{i+j})/(x_i^c - x_{i+j}) \tag{1.7}$$

where R_0 is the external radius of the rod at the point of fixing to the spar, it can be written in the form

$$G_{ij}^0 = (-1)^j \frac{\rho p_1^0(x_i^*)P_2}{2\sigma R_0}(x_i^c - x_{i+j})^3 \tag{1.8}$$

where x_i^* is a value between x_i^c and x_{i+j}. Denoting the greatest of the lengths of the rods by l_c and using the definition of an integral, it is possible to estimate the total mass of the rods participating in the formation of the upper surface of the wing. We obtain

$$G^0 = \Sigma\ (G_{i0}^0 + G_{i1}^0) \leqslant \frac{\rho P^0 l_c^2}{2\sigma R_0}; \quad P^0 = P_1^0 P_2, \quad P_1^0 = \int_0^b p_1^0(x)dx \tag{1.9}$$

where P^0 is the total load on the upper surface of the wing.

The total mass of the rods which are fastened to spar i as viewed from spar j to form the lower surface of the wing has a form which is analogous to (1.8) and is obtained by replacing $p_1^0(x_i^*)$ by $p_1^1(x_i^{**})$, where x_i^{**} is a value between x_i^c and x_{i+j}. For the total mass G_1 of the rods participating in the formation of the lower surface of the wing, we obtain an estimate which has the form of estimate (1.9) after the quantity P^0 in it has been replaced by the quantity P^1 for the total load on the lower surface of the wing.

Since, in order to reduce the mass of the wing, it is advantageous to increase the number of rods which take up the distributed load, we assume that

$$y_k = 2R_0(k-1), \quad k = 1,\ldots,m_i, \quad m_i = l/(2R_0)$$

which corresponds to the case when neighbouring rods touch. Taking account of this, we estimate the mass of the strips which produce the upper surface of the wing. We assume that $p_2(y) = p_2^* = \text{const}$ and we shall presume that each strip which is joined to a rod which is fastened to spar i as viewed from spar j is formed as a cantilever of equal strength over the surface on which the pressure is distributed

$$p_{ij}^0 = \frac{p_2^*}{x_i^c - x_{i+j}} \int_{x_{i+j}}^{x_i^c} p_1^0(x)dx \tag{1.10}$$

We will find the greatest height of a strip h_{ij}^0 using the stability condition for a cantilever of length $b_1/2 = R_0$ with a rectangular cross-section of a certain width b_2 and variable height h, that is, we use the condition $\sigma = 6M_{ij}^0/[b_2(h_{ij}^0)^2]$, where $M_{ij}^0 = p_{ij}^0 b_2 R_0^2/2$ is the bending moment. It follows from this that $h_{ij}^0 = R_0(3p_{ij}^0/\sigma)^{1/2}$. On taking account of the fact that the mean height of a strip is $h_{ij}^0/2$, we write the expression for the total mass of the strips forming the upper surface of the wing as

$$G' = \frac{\rho l R_0}{2}\left(\frac{3}{\sigma}\right)^{1/2} \sum_{i=1}^{m} \sum_{j=0}^{1} (p_{ij}^0)^{1/2}(-1)^j(x_i^c - x_{i+j}) \tag{1.11}$$

Then, using formula (1.10), we find an estimate of this mass

$$G' \leqslant \rho R_0(3P^0 bl\sigma^{-1})^{1/2}/2 \tag{1.12}$$

The total mass G'' of the strips forming the lower surface of the win has an estimate which is obtained from (1.12) by replacing P^0 by P^1.

Note that the parameter R_0 and the length of the conical rods which are fastened to the spars can be chosen so that the mass of the spars G^* are several times greater than the mass of the rods (G^0 + G^1) fastened to them and, in its turn, this mass will be several times greater than the mass of the strips forming the surface of the wing, (G' + G'').

2. VIBRATION OF THE WING

We shall assume that each rod participating in the formation of the wing skin is designed on the assumption that the load which is taken up by it is uniformly distributed along its length. In particular, the linear load on a rod k, which is fastened to a spar i as viewed from spar j to form the upper surface of the wing, is assumed to be as follows:

$$q_{ijk}^0 = \int_{y_k}^{y_{k+1}} p_2(y)dy \frac{1}{x_i^c - x_{i+j}} \int_{x_{i+j}}^{x_i^c} p_1^0(x)dx$$

then

$$\sigma = M_{ijk}^0(x)/[\pi f_{ij}^2(x)\delta_{ijk}^0], \quad M_{ijk}^0(x) = q_{ijk}^0(x - x_{i+j})^2/2$$

($M_{ijk}^0(x)$ is the bending moment in a section around a certain point (x, y_k^c) of the rod axis). Furthermore, in the case of the above-mentioned assumption, the total mass of the rods which are fastened to spar i as viewed from spar j is described by the sum of two expressions, the first of which has the form of (1.8) in which

$$p_1^0(x_i^*) = \frac{1}{x_i^c - x_{i+j}} \int_{x_{i+j}}^{x_i^c} p_1^0(x)dx$$

and the second expression is obtained from the first on replacing $p_1^0(x)$ by $p_1^1(x)$.

When account is taken of this, assuming that

$$p_1^j(x) = a_j p_1(x)$$

where a_j (j = 0, 1) are the chosen constants, we have the following expression for the distribution of the mass of the rods, which are fastened to the spar i, along its axis

$$g_i^*(y) = u(y)\left[(x_i^c - x_i)^2 \int_{x_i}^{x_i^c} p_1(x)dx + (x_{i+1} - x_i^c)^2 \int_{x_i^c}^{x_{i+1}} p_1(x)dx\right]$$

where

$$u(y) = (a_0 + a_1)\rho p_2(y)/(2\sigma R_0) \tag{2.1}$$

The expression $g_i^*(y)$ can be approximately replaced by the following

$$g_i(y) = u(y)\frac{(x_{i+1} - x_i)^2}{4} \int_{x_i}^{x_{i+1}} p_1(x)dx \tag{2.2}$$

We will estimate the error arising from this replacement, assuming that the function $p_1(x)$ is linear in the interval $[x_i, x_{i+1}]$: $p_1(x) = \bar{p}_1 + p_i'x$ where $p_i' \neq 0$ (in the case when p_i' = 0, there is no error). After some calculations, we obtain

$$\frac{g_i^*(y) - g_i(y)}{g_i(y)} = \frac{(x_{i+1} - x_i)^4}{27(x_{i+1} + x_i + 2\bar{p}_i / p_i')^4} \tag{2.3}$$

It is clear that the greatest error corresponds to the end spar. On substituting the values $\bar{p}_i$ = 0 and x_i = 0 for i = 1 into expression (2.3), we find that this error does not exceed 4%. If account is taken of the fact that the form of vibration of a wing spar depends on the sum of the total linear masses of the spar and the rods which are fastened to it, the relative error is even smaller. Allowing for this, we shall use formula (2.2) for the distribution of the mass of the rods linked with spar i.

We assume that the rod parameters are chosen such that the mass of the strips joined to them is negligibly small compared with the mass of the rods and spars so thall not take account of the mass of the strips in treating the wing vibration.

Using formulae (1.2) and (1.3), we write the moment of inertia of a cross-section of a spar i in the form

$$J_i(y) = I_i I(y) f(y)/\sigma \tag{2.4}$$

and write the mass per unit length of spar i, taking account of (1.4) and (1.2), in the form

$$\rho F_i(y) = 2\rho I_i I(y)/[\sigma f(y)] \tag{2.5}$$

Assuming that

$$x_i = (i-1)l_*, \quad l_* = b/m, \quad i = 1,\dots,m \tag{2.6}$$

we have from (2.2) an expression for the mass distribution of the rods

$$g_i(y) = u(y) l_*^2 I_i / 4 \tag{2.7}$$

The centre of mass of a section of the wing is the point (x_g, y) with the coordinate (henceforth summation is carried out from $i = 1$ to $i = m$)

$$x_g = \Sigma[\rho F_i(y) + g_i(y)] x_i^c / \Sigma[\rho F_i(y) + g_i(y)]$$

from where, on taking account of (2.5) and (2.7), we have $x_g = \Sigma I_i x_i^c / \Sigma I_i$. By the centre of rigidity of a cross-section of the wing we mean [3] the point (x_*, y) with coordinate

$$x_* = \Sigma EJ_i(y) x_i^c / [\Sigma EJ_i(y)]$$

where E is the modulus of elasticity. Then, on taking account of (2.4), we conclude that $x_* = x_g$. Hence, if all the spars have the same initial flexure, then, after the load has been removed, the wing will execute free bending vibration in a vacuum, described by the equation [3]

$$\frac{\partial^2}{\partial y^2}\left[E\Sigma J_i(y)\frac{\partial^2 z}{\partial y^2}\right] + \Sigma[\rho F_i(y) + g_i(y)]\frac{\partial^2 z}{\partial t^2} = 0 \tag{2.8}$$

(t is the time). On taking account of (2.4), (2.5) and (2.7) and dividing by ΣI_i, we have

$$\frac{E}{\sigma}\frac{\partial^2}{\partial y^2}\left[I(y)f(y)\frac{\partial^2 z}{\partial y^2}\right] + \left[\frac{2\rho I(y)}{\sigma f(y)} + \frac{l_*^2 u(y)}{4}\right]\frac{\partial^2 z}{\partial t^2} = 0 \tag{2.9}$$

The free vibration of each spar is described in this manner. We assume that the wing spars are all fixed in the same way. Hence, for the same initial form of flexure, they will execute the same vibration in a vacuum, which does not lead to any distortion and twisting of the wing profile.

We shall now dwell on the bending and vibration in an air stream. We shall assume that the expected load distribution of the wing in the case of an angle of attack α and a dynamic pressure q is described by the product $p_{\alpha q}(x)p_2(y)$, where $p_{\alpha q}(x) = c(\alpha)\varphi(q)p_1(x)$, $c(\alpha)$ and $\varphi(q)$ are continuous functions, $q = \rho_0 V^2/2$, ρ_0 is the air density and V is the air flow density. When account is taken of this, we can write the bending moment in a section passing through the point (x_i^c, y) of the axis of spar i in the form

$$M_{i\alpha q}(y) = c(\alpha)\varphi(q)M_i(y)$$

where $M_i(y)$ is the calculated bending moment (1.2). Hence, the curvature of the spar axis which is described by the formula [2]

$$\varkappa_{i\alpha q}(y) = M_{i\alpha q}(y)/[EJ_i(y)]$$

is transformed using (1.3), to the form

$$\varkappa_{i\alpha q}(y) = c(\alpha)\varphi(q)\sigma/[Ef(y)]$$

Consequently, the initial form of the bending is the same for all the spars.

In estimating the vibration of the wing, we do not take its basic load, which is counterbalanced by elastic forces, into consideration. We do take account of the additional load which is a consequence of the additional deformations of the wing accompanying the vibration. Since the road per unit length on a spar i is $q_{i\alpha q}(y) = c(\alpha)\varphi(q)p_2(y)I_i$ and it is applied at the point (x_i^c, y) of its axis, the load per unit length on the whole wing is applied at the point (x_p, y) (the focus of the wing) with coordinate

$$x_p = \Sigma q_{i\alpha q}(y)x_i^c / \Sigma q_{i\alpha q}(y)$$

and this point coincides with the centres of rigidity and mass. Vibration caused by a random flexural pulse will therefore remain purely bending vibration and the increment in the angle of attack α due to the deformation has the form [3] $\Delta\alpha = -V^{-1}\partial z/\partial t$. The corresponding increment in the coefficient $c(\alpha)$ is

$$\Delta c(\alpha) = [\partial c(\alpha)/\partial\alpha](-V^{-1}\partial z/\partial t)$$

On taking account of the load per unit length which has appeared due to the deformation, then using (2.4), (2.5) and (2.7) and cancelling by ΣI_i, we have an equation which differs from (2.9) on the right-hand side and now has the form

$$p_2(y)\varphi(q)\Delta c(\alpha)$$

All the spars in the air flow will execute the same bending vibration, subject to this equation, and there will be no flutter.

3. THE CASE OF A DEVIATION OF THE ACTUAL LOAD DISTRIBUTION FROM THE EXPECTED LOAD DISTRIBUTION

We shall assume that the actual load distribution at an angle of attack α and a dynamic pressure q is described by the product $p^*_{\alpha q}(x)p_2(y)$, where the function $p^*_{\alpha q}(x)$ is not completely identical to the function $p_{\alpha q}(x)$ which characterizes the expected load distribution over the profile for the same α and q. To take this into account, it is necessary to construct a wing such that the elements of the wing skin fastened to a single spar partially overlap the elements of the wing skin fastened to a neighbouring spar (Fig. 4). On the overlapping segments of the wing skin, which are facing the air flow and associated with some single spar, they will either independently take up the load or they will transfer the load to the overlapping elements of the wing skin associated with a neighbouring spar.

In this case, it is necessary to design each spar i and the elements of the wing skin which are fastened to it for the perception of a load with the wider segment included between the planes $x = x_i^0$ and $x = x^1_{i+1}$, where

$$x_i^0 = x_i - (i-1)\Delta_0 / m, \quad x^1_{i+1} = x_{i+1} + \Delta_0(m-i)/m \tag{3.1}$$

the parameters x_i ($i = 1, \ldots, m$) have the values (2.6), and by Δ_0, we mean the width of the overlap segment between neighbouring spars. Here, in formulae (1.1) and (1.2), which determine the position of the axis of a spar i and the calculated bending moment in its section, it is necessary to replace x_i by x_i^0 and x_{i+1} by x^1_{i+1}. Allowing for this, in the second of these formulae, we replace the integral I_i by the integral

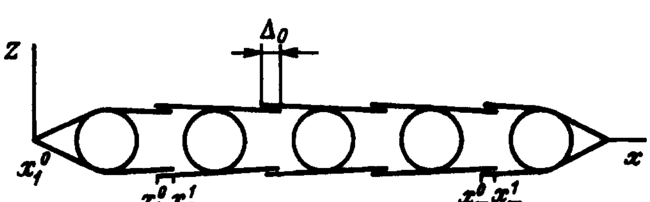

Fig. 4.

$$I_i^+ = \int_{x_i^0}^{x_{i+1}^1} p_1(x)dx, \quad i = 1, \ldots, m$$

which, using relations (3.1) and (2.6) is approximately expressed as

$$I_i^+ \approx I_i + \left[p_1(x_i)\frac{i-1}{m-1} + p_1(x_{i+1})\frac{m-i}{m-1} \right]\frac{m-1}{m}\Delta_0 = I_i + p_{1i}l_* w \tag{3.2}$$

where p_{1i} is the value of the function $p_1(x)$ in the case of such as $x \in [x_i, x_{i+1}]$ for which the equality (3.2) is satisfied and

$$w = (m-1)\Delta_0/b \tag{3.3}$$

is the relative magnitude of the overlap. On allowing for the fact that $\Sigma p_{1i}l_*$ is the approximate value of the integral of $p_1(x)$ over the interval $[0, b]$, we obtain

$$G_* = G^*(1+w) \tag{3.4}$$

for the total mass of the spars, where G^* is expression (1.5) or, if the spars are conical, expression (1.6).

In this case, the approximate expression (2.2) for the mass distribution of the rods which are fastened to a spar i takes the form

$$g_i(y) = u(y)l_0^2 I_i^+/4 \tag{3.5}$$

where $l_0 = x_{i+1}^1 - x_i^0$ or, on taking account of expressions (3.1) (2.6) and (3.3), we have

$$l_0 = [b + (m-1)\Delta_0]/m = b(1+w)/m$$

Using relations (2.1) and (3.2) and integrating $\Sigma g_i(y)$ with respect to y from 0 to l, we obtain an expression for the total mass of the rods linked with the spars in the form

$$G_c = (a_0 + a_1)\rho P b^2(1+w)^3/(8\sigma R_0 m^2) \tag{3.6}$$

On estimating, in the case under consideration, the total mass of the strips which form the upper surface of the wing, it is necessary to replace x_{i+j} by x^j_{i+j} in formulae (1.10) and (1.11). Then, by using these formulae and taking account of the fact that, by virtue of (3.1) and (3.3)

$$\Sigma(x_{i+1}^1 - x_i^0) = b(1+w)$$

and, also, using equality (3.2) and the relation $p_1^0(x) = a_0 p_1(x)$, we obtain an estimate for the total mass of the strips forming the upper surface of the wing

$$G' \leqslant \rho R_0 (3a_0 Pbl\sigma^{-1})^{1/2}(1+w)/2 \tag{3.7}$$

The total mass G'' of the strips forming the lower surface of the wing has a limit which is obtained from limit (3.7) by replacing a_0 by a_1.

We will now deal the vibration of the wing. In the case under consideration, the formulae obtained from (2.4) and (2.5) by replacing I_i by I_i^* hold. Furthermore, formula (3.5) holds. As a consequence of this, the centre of rigidity and the centre of mass are at the same point (x_*, y), where $x_* = \Sigma I_i^+ x_i^c/\Sigma I_i^+$. Hence, the equation of the free bending vibration of a wing in a vacuum (2.8) which, after using the above mentioned formulae and cancelling out ΣI_i^+, takes a form similar to (2.9), holds.

It follows from this equation that all the spars will execute the same free vibration in the case of the same initial form of flexure. The initial flexure of the spar is the same if the curvature of the axis $\varkappa_i(y) = M_i^*(y)/[EJ_i(y))]$, where $M_i^*(y)$ is the bending moment in a section of the spar i, is the same in the case of any ith spar. Here, using formula (2.4) with I_i replaced by I_i^+ and introducing $M_i^*(y) = I_i^* I(y)$, we note that the function $\varkappa_i(y)$ is the same for all spars if the ratio I_i^*/I_i^+ $(i = 1, \ldots, m)$ is constant, where I_i^*, taken with the coefficient $p_2(y)$, characterizes the fraction of the load per unit length on the wing which is taken up by a spar i. Hence, the condition for identical bending of the spars has the form

$$I_i^* = I_i^+ \Sigma I_i^* / \Sigma I_i^+, \ i = 1,\ldots,m, \quad \Sigma I_i^* = \int_0^b p_{\alpha q}^*(x)dx \tag{3.8}$$

The wing load per unit length on the segment, where the wing skin elements, linked with the wing spars i and $(i + 1)$, overlap, is $p_2(y)I_i^-$, where

$$I_i^- = \int_{x_{i+1}^0}^{x_{i+1}^1} p_{\alpha q}^*(x)dx, \quad i = 1,\ldots,m-1$$

The relative fraction of β_i from this load taken up by the spar $(i + 1)$, is found using (3.8) from the equalities

$$I_i^* = \int_{x_i^0}^{x_{i+1}^1} p_{\alpha q}^*(x)dx - \beta_i I_i^- - (1-\beta_{i-1})I_{i-1}^- \quad (\beta_0 \equiv 1)$$

$$i = 1,\ldots, m-1$$

We assume that $\beta_i \in [0, 1]$ $(i = 1, \ldots, m - 1)$ and a $p_{\alpha q}^*(x) \leqslant p_1(x)/\eta_p$, where η_p is the safety factor. Then, in calculating the wing skin elements participating in the formation of the lower wing surface, we can take $a_1 = 1$, thereby equating the calculated load on the lower wing surface to the total calculated load.

In considering the vibration of the wing in an air flow, we introduce $p_{\alpha q}^*(x) = p_q^*(x)c(\alpha)$, where $c(\alpha)$ is a function of the angle of attack α. Since the load per unit length (x_i^c, y) on a spar i is applied at the point (x_i^c, y) of its axis, the load on the whole wing is applied at a point with a coordinate $x_p = \Sigma I_i^* x_i^c / \Sigma I_i^*$, and this point, by virtue of (3.8), coincides with the centre of mass and the centre of rigidity. Hence, the bending vibration of the wing will not be transformed into bending-torsional vibration and is described by an equation which can be obtained if, in an equation of the form of (2.8), account is taken of the air load caused by the deformation and use is made of expressions (2.4) and (2.5) with I_i replaced by I_i^+ and expression (3.5) after division by ΣI_i^+. An equation is obtained which differs from (2.9) in that l_0 is on the left-hand side instead of I_* and the right-hand side has the form

$$p_2(y)\Delta c(\alpha)c^{-1}(\alpha)\Sigma I_i^* / \Sigma I_i^+$$

All the spars will execute the same vibration, which does not lead to flutter.

4. CONCLUDING REMARKS

We wish to emphasize certain special features of the construction of the wing. The external form of the spars is assumed to be the same but the thickness of the end sections of each spar is proportional to the load $p_2(y)I_i^+$ which it takes up and, if the external shape is close to conical and the load distribution over the wing span is taken to be uniform, the thickness of the cross-section of a spar is constant along its axis. The thickness of the wing profile is close to the external diameter of the spars in the section being considered. Note that, if spars with a different external diameter are used in the wing profile then, in order to ensure that the spars bend to the same extent, it is necessary to increase their mass, which becomes greater than the mass necessary to ensure stability. To control the motion of an aircraft, it is advisable to tilt the spars in the plane of the wing with the possibility of: (1) changing the taper of the wing and, as a consequence, changing the area of the wing and the magnitude of the lift—to control the banking moment, (2) to change the sweepback of the wing, that is, to shift the point of application of the lift force along the axis of the fuselage—to control the pitching moment.

We will now present an example of an estimate of the mass of a wing which employs the proposed design scheme using the characteristics of a Yak-18 aircraft: take-off weight $P_b = 1100$ kg, design load factor coefficient $n_p = 13.5$, wing area $S = 17$ m^2, aspect ratio $\lambda = 6.61$, relative thickness $\bar{c}_0 = 0.15$, taper $\eta = 2$ and fuselage diameter $d_0 \approx 1$ m.

We find the following additional characteristics: them ean chord of the wing $b_c = (S/\lambda)^{1/2} \approx 1.6$ m, the length of the wing $l = (\lambda b_c - d_0)/2 \approx 4.8$ m, the chord on the axis of the fuselage $b_0 = 2\eta b_c/(\eta + 1) \approx 2.13$ m, the chord of the wing at the fixing point $c_0 = b\bar{c}_0 \approx 0.305$ m and the design load on the wing $P = n_p(P_b/2)(S - b_0 d_0)/S \approx 6500$ kg.

We will now estimate the mass of a wing designed using the proposed scheme with the values of the parameters l, b, c_0, P indicated above. The wing is assumed to be rectangular and, correspondingly, has a 33% greater area than a wing of a Yak-18 aircraft. We assume that the load distribution is uniform over the wing span: $p_2(y) = \text{const}$. It is assumed that the spars are conical with a radius at the point of attachment $f(0) = 0.145$ m. On taking the number of spars as $m = 4$, and the width of the overlap segment as $\Delta_0 = 0.05$ m, we find, using formula (3.3), the value $w = 0.0739$. Then, using formulae (3.4) and (1.6), we find the total mass of the spars $G_* = 25.5$ kg, assuming that the alloy B95T is used as the wing material for which $\rho = 2850$ kg/m^3 and $\sigma = 62 \times 10^6$ kg/m^2. We next assume that the rods which are fastened to the spars are conical with a radius at the point of attachment $R_0 = 0.005$ m. We shall take the coefficients which characterize the rated load on the upper and lower surfaces of the wing as $a_0 = 0.6$, $a_1 = 1$. Then, according to formula (3.6), the mass of the rods is $G_c = 3.81$ kg. Next, using formula (3.7) and replacing a_0 in this formula by a_1, we find for an estimate of G'' that the mass of the strips which form the wing surface is $G' + G'' \leqslant 0.75$ kg. Hence, the total mass of the two wings in the proposed scheme is $2(G_* + G_c + G' + G'') \approx 60$ kg. Note that the mass of the wings of a Yak-18 aircraft is significantly greater [4].

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